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Collisional evolution of mass-distribution spectrum of planetesimals

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Abstract—The early growth of planetesimals by mutual direct collisions is numerically simulated with a Monte Carlo technique to show how planetesimals with specific mass and velocity distributions evolve into a full-size planet. Four types of collisions are taken into into account: rebound, erosion, catastrophic break-up, and coagulation. It is shown that evolution of mass-distribution spectrum is expressed by an inverse power relation such as $N(m, t) = c(t)m^{-\alpha}$, where $N(m, t)$ is the cumulative number of bodies with mass larger than m. α is about 0.7 when coagulation and fragmentation compete. It is also suggested that whether or not a planetesimal can survive catastrophic collision is primarily dependent on mean relative velocity and mechanical properties of planetesimals. It is necessary for the early growth of rocky (basaltic) materials that mean relative velocity be much smaller than 0.1 km/s. Otherwise we need to introduce something like nucleating agents (such as iron bodies which have plastic properties at temperatures higher than 200°K) for the formation of terrestrial planets.

INTRODUCTION

The mass distribution of impacting bodies and the time-scale of accumulation crucially affect the early thermal state of planets and satellites (Mizutani *et al.*, 1972; Wetherill, 1976; Safronov, 1978). Therefore, the clarification of planetary accretion process is very important in this context. Many studies so far reported have assumed that planetesimals coagulate upon collision (Safronov, 1972; Mizutani *et al.*, 1972; Weidenschilling, 1974). It is, however, more reasonable to assume that coagulation and fragmentation of planetesimals compete in the early accretion stage and that a body surviving catastrophic break-up becomes a full-size planet. In this study, we present a Monte Carlo simulation of a direct collision process of planetesimals with radii of several km taking into account coagulation and fragmentation simultaneously. The study is intended to clarify the effects of relative velocity distribution and mechanical properties of planetesimals on the collisional evolution of planetesimals.

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COLLISION TYPES

Collision between two planetesimals can have three possible fates: (i) rebound, (ii) catastrophic disintegration, and (iii) coagulation.

(i) Rebound: If the stress generated by an impact is lower than the elastic limit, bodies *rebound* from each other with a relative velocity of k v_{imp} where k is a coefficient of restitution and v_{imp} is the impact velocity. The minimum impact velocity, V_B, which generates a stress corresponding to the elastic limit is dependent on material properties. It is estimated to be about 0.03 km/s for spheric basaltic rocks (Matsui and Mizutani, 1977; Hartmann, 1978). If the impact velocity is greater than V_B, but is not large enough to disintegrate either projectile or target bodies, the result may be rebound with fragmentary chips (Greenberg *et al.*, 1977).

(ii) Catastrophic disintegration: If the kinetic energy liberated at collision is large enough to disintegrate completely either the projectile or the target body, the result is a *castastrophic disintegration* of either or both bodies. Here projectile and target body represent smaller and larger body respectively. This means that fragments are ejected with finite velocities. The critical velocity, V_c, subdividing castastrophic disintegration and rebound with chips regimes can be estimated to be

$$\sqrt{2E_{rup}\left(\frac{m_t + m_p}{m_p}\right)}$$

if all the kinetic energy liberated at collision is converted to the energy required for catastrophic rupture, where E_{rup} is the minimum catastrophic rupture energy per gram and m_t and m_p are target and projectile masses. According to Fujiwara *et al.* (1977), E_{rup} is about 10^7 erg/g for basaltic rocks. Hence, for basaltic body, V_c is about 0.063 km/s if projectile and target masses are the same. In the present study, it is assumed that the minimum rupture energy for weaker bodies is $^1/_{100}$ times that of a basaltic body, or $V_c \sim 0.0063$ km/s. These values agree well with values found by Hartmann (1978) for non-equal size bodies. At impact velocities larger than V_c there occurs disintegration of either the projectile or both the projectile and the target bodies depending on their mass ratio. If the liberated kinetic energy is large enough to disintegrate the projectile body, but not large enough to fragment the target body ($m_p E_{rup} < \frac{1}{2} m_p v_{imp}^2 < m_t E_{rup}$), there occurs *erosion* of the target body (that is, the ejected total mass is larger than the projectile mass). All estimates mentioned above are applicable to brittle material. However, we need a different treatment for ductile material such as iron (Matsui and Mizutani, 1977).

(iii) Coagulation: The most important collision type for the early growth of planetesimals is, of course, a *coagulative* collision. This occurs either when rebound velocity, v_{reb} ($=k\ v_{imp}$), is smaller than the escape velocity of the colliding body system, v_e, or when the total ejected mass of the fragments with velocities greater than v_e is smaller than the projectile mass. The second condition is very dependent on the velocity distribution of ejecta which is poorly known so far. However, using the impact experiment by Gault *et al.* (1963), the velocity distribution of ejecta can be approximated by

$$\begin{aligned} G\ (v_e, v_{imp}) &= 1,\ V_f > v_{imp} > v_e \\ &= 0,\ v_e > 1/\beta\ v_{imp} \\ &= \frac{V_f^2\ v_e^{-2} - \beta^2 V_f^2\ v_{imp}^{-2}}{1 - \beta^2 V_f^2\ v_{imp}^{-2}}, \\ &\quad 1/\beta\ v_{imp} \geq v_e \geq V_f, \end{aligned} \tag{1}$$

where G is the fraction of ejecta with velocity greater than v_e, V_f is a critical velocity subdividing $G = 1$ and $G < 1$, β is the inverse of coefficient of restitution k_2 for rebound-with-chips and v_{imp} is the impact velocity given by

$$v_{imp}^2 = v_e^2 + v_\infty^2, \tag{2}$$

where v_∞ is the relative velocity between the target and the projectile bodies at infinity (i.e. before their mutual gravitation comes into play). $G = 1$ means that all debris can escape from colliding pairs, and $G < 1$ means that some portion of debris can escape from colliding pairs. Therefore mass gain of the target body would occur if $0 \leqq G < 1$. According to the impact experiments by Gault *et al.* (1963), V_f is 0.05 km/s for basaltic body in the erosive (cratering) case. It is a future problem to get the value of V_f.

Calculation Technique

Application of a Monte Carlo technique to the accretion of bodies was first developed by Dodd *et al.* (1972). They studied coagulative and disruptive cases separately. In the present study, however, we apply this technique to the case in which both coagulation and fragmentation are simultaneously taken into account (Matsui, 1977). We assume that (1) the ensemble of colliding bodies occupies a fixed volume; (2) only two-body collisions occur; (3) as a result of collision, there occurs rebound, accumulation, erosion or fragmentation; (4) particles and their fragmentas are spherical and have the same material density; (5) the masses of the particles lie in a specified range, $M_O \leq m < M$, and (6) fragments with masses less than M_O are immediately removed from the system and do not take part in subsequent collisions.

The first step is to compute the collision probability of any pair of particles in the system using a particle-in-box approximation. Let n_i represent the number of particles with mass m_i. Defining σ_{ij} to be the collision probability between any pair of masses (m_i, m_j) in unit time interval, we have

$$\sigma_{ij} \propto n_i\, n_j\, (m_i^{1/3} + m_j^{1/3})^2 \left(1 + \gamma \frac{m_i + m_j}{m_i^{1/3} + m_j^{1/3}}\right), \quad i \neq j$$

$$\sigma_{ii} \propto 2\, n_i\, (n_i - 1)\, m_i^{2/3}\, (1 + \gamma\, m_i^{2/3}) \qquad i = j \tag{3}$$

where

$$\gamma = \frac{2G'\, M_o^{2/3}}{v_{ij}^2\, (3/4\, \pi\, \rho)^{1/3}},$$

G', v_{ij} and ρ are the gravitational constant, relative velocity and mass density of bodies. Because of the restriction of computer time, v_{ij} is assumed to be equal to the r.m.s. relative velocity when we calculate σ_{ij}. However, a random number generated in the computer is used as the relative velocity when we judge the collision types. Collision probability line is given by summing the individual collision probabilities (see Fig. 1). As the individual collision probability is normalized by the total collision probability (i.e. $\sigma_{total} = \sum_i \sum_j \sigma_{ij}$), the length of collision probability line is equal to 1 and the length of each segment comprising the line is proportional to $\sigma_{ij}/\sigma_{total}$. Then a random number between 0 and 1 is generated. The segment on the line assigned by the random number gives the particle pair which collide. The waiting time, t, between collisions is given by $t = 1/\sigma_{total}$. The time variable is incremented to give a measure of the time elapsed since the beginning of the process.

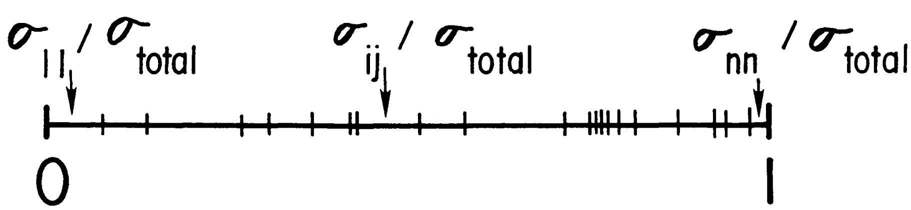

Fig. 1. Collision probability line.

Once the collision pair is chosen, we decide on a type of collision following the criteria defined before. Mass distribution of debris caused by rebound with chips and catastrophic disintegration of either the projectile or both the projectile and the target bodies is assumed to obey the comminution relation expressed by $f(m)\, dm = c,\ m^{-q}dm$ $(q = 1.8)$. In the numerical simulation, we cannot use a continuous function, so that we replace the above power function by a discrete function. The mass m_i of the body in the mass interval (M'_i, M'_{i+1}) is given by

$$m_i = \int_{M'_i}^{M'_{i+1}} m \cdot f(m)\, dm$$

$$= (\frac{q-1}{2-q}\, i\, M'_o + 1)^{2-q/1-q} - \{\frac{q-1}{2-q}(i+1)\, M'_o + 1\}^{2-q/1-q} \quad (i = 0, 1, 2, \ldots\ldots, n) \qquad (4)$$

where M'_O is the upper limit of mass range.

Collisional Evolution of Planetesimals

Model: The outcome of collision is generally dependent on impact velocity, mechanical properties (such as mechanical strength, shape and surface properties), escape velocity of the bodies, and their mass ratio (Matsui and Mizutani, 1977; Hartmann, 1978). Hence, the parameters defining the models are those related to these quantities. A wide variety of initial conditions are studied. We assume the relative velocity distribution to be given by a 2-dimensional Maxwell-Boltzmann type distribution,

$$f(v)\, d\, v = (\pi/2\, C^2)\, e^{-(\pi v^2/4\, C^2)} v dv, \qquad (5)$$

where C is mean relative velocity, and mean relative velocity is assumed constant with time. The second assumption might be reasonable if the mass distribution of planetesimals varies over a time-scale smaller than the time required for the balance of dispersion and damping of relative velocities. This is very likely to be the case as will be shown later. To clarify the effect of mechanical properties, we studied two cases: one model with basaltic bodies and the other with weaker bodies. The surface of the bodies is assumed to be smooth and not to be covered by regolith. Mechanical properties of basalt and weaker bodies are summarized in Table 1. The coefficient of restitution, k, is different in the pure rebound and the rebound-with-chips cases, particularly for weaker materials.

Numerical results: In Fig. 2 is shown the collisional evolution of mass-distribution spectrum of planetesimals. The ordinate is the cumulative number of bodies with mass larger than m. This model is characterized as follows: the bodies are initially all of the same mass, M^* ($= 10^{17}$g), their mechanical properties are similar to the weaker material ($V_b = 0.003$ km/s, $V_c = 0.0063$ km/s, and $V_f = 0.005$ km/s), and the mean relative velocity, $\overline{v}_\infty = 2\, V_e$ where V_e is the escape velocity of the largest body at $t = 0$ ($\sim$0.0026 km/s). Thus the initial mass-distribution spectrum is a step-function. Numbers in the figure represent the total number of collisions which is nearly proportional to the elapsed time. Within a very short period ($\sim 50\tau_{col}$) there occurs transfer of mass. About 60% of bodies can grow over M^* and the others are fragmented. Then mass-distribution approaches the distribution expressed by an inverse power relation. After 5780 collisions, all bodies with mass larger than M_O (smallest mass range) are collected by one largest body which incorporates about 67% of

TABLE I. MECHANICAL PROPERTIES OF PLANETESIMALS

Model	k_1*	k_2*	V_b**	V_c**	V_f**
Basalt	0.85†	0.8†	0.03†	0.063††	0.05†††
Weaker Body	0.5†	0.1†	0.003†	0.0063	0.005

*k_1 and k_2 are coefficients of restitution for pure rebound and rebound with chips cases, respectively.

**Critical velocities related to collision types; V_b subdividing pure rebound and rebound with chips regimes, V_c subdividing rebound with chips and catastrophic disintegration regimes, and V_f subdividing $G = 1$ and $G < 1$ where G is a fraction of ejecta with velocities greater than escape velocity of the colliding pairs. Unit is km/s.

†based on the experimental data by Hartmann (1978).

††based on the experimental data by Fujiwara *et al.* (1977).

†††based on the experimental data by Gault *et al.* (1963).

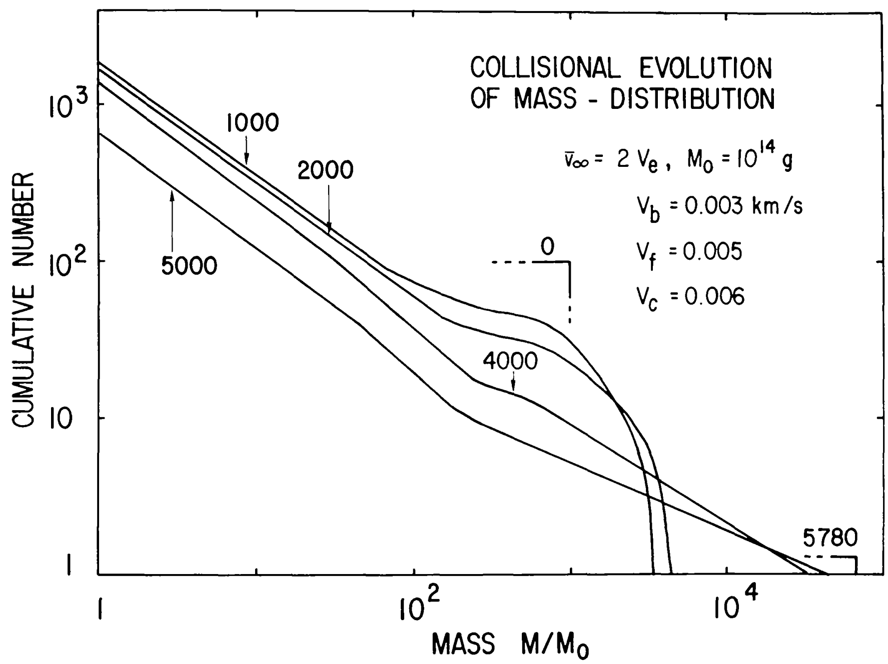

Fig. 2. Collisional evolution of mass-distribution spectrum. This model is characterized as follows: δ - function type initial mass distribution, mechanical properties are similar to weaker materials, and $\bar{v}_\infty = 2V_e$. The ordinate is cumulative number of the bodies with the mass larger than m. Numbers shown in the figure represent the total number of collisions. Initial mass distribution is shown by the line with 0. Bodies corresponding to about 67% of the initial total mass are transferred to the mass range larger than the initial mass. Slopes for the smaller mass range are in the narrow range around - 0.71.

the initial total mass, and other bodies (corresponding to about 33% of the initial total mass) are transferred to the mass range smaller than M_O. Hence, it is clear that growth of planetesimals occurs in this model. Damping time scale due to inelastic collision is about 100 τ_{col} in this case, so that growth time ($\sim$50 τ_{col}) is faster than this. Growth time from m = M* to m = 0.97 M_{final} is about 700 τ_{col}, where M* and M_{final} are initial mass of bodies and final mass of the largest body respectively, and τ_{col} is the mean collision interval at t = 0 (=4 ρ R/3 ρ_sv, where ρ and ρ_s are the material and space densities, R is radius of the body and v is dispersion velocity). In real time τ_{col} may be about 1 year at 1 A.U. if the present earth's mass is used as the total mass of bodies, and if bodies are distributed in a ring with radial interval from 0.85 A.U. to 1.23 A.U. Even for the model with lower mean relative velocity ($\bar{v}_\infty = V_e$) the growth time is about the same as in this model. However, in this case almost all bodies can grow beyond M* and finally the largest planetesimal incorporates about 95% of initial total mass. The other interesting feature of this figure is a constant slope ($\sim$ −0.71) at the smaller mass range during the period from 1000 to 4000. Total number of bodies larger than M_O is also almost constant during this period. This suggests that decrease and increase in number of bodies due to both fragmentation and coagulation are balanced (it means $\partial n(m, t)/\partial t = 0$, where n(m,t) is incremental number of bodies within the mass range from M_O to $\sim$100M_O). Therefore evolution of mass-distribution spectrum is expressed by $N(m,t)= c(t)\ m^{-\alpha}$, where N is cumulative number, $\alpha = 0.71$. It is shown to be common in all models where coagulation and fragmentation are competing irrespective of initial mass-distribution that the evolution is expressed by the above equation, ($\alpha = 0.7 \pm 0.05$) (Matsui, 1977).

The mean relative velocity, $\bar{v}_\infty$, obviously affects the collisional evolution of planetesimals. Figure 3 shows the evolution of the model with higher mean relative velocity ($\bar{v}_\infty \sim 5\ V_e$). The other parameters are the same as the previous model. It is seen that a few bodies can grow beyond M* but finally all bodies (100%) are transferred to the mass range smaller than M_O. Slope in the smaller mass range also seems to be constant with time and is around −0.82.

In the case that the impact velocity is close to the critical velocity, V_c subdividing rebound-with-chips and catastrophic-disintegration regimes, whether or not planetesimals can survive catastrophic disintegration depends on V_f. Hence, the relation between V_c and V_f plays a key role in the growth process beyond the mass, M_c, whose escape velocity is V_c ($M_c \sim 1.46 \times 10^{18}$g for this model). For example, $V_c \ll V_f$ is the favorable condition from the point of view of accretion. In Fig. 4 is shown the evolution of the model whose V_f is equal to V_c, all other parameters being the same as in the first model. In this model coagulation and fragmentation compete through collisional evolution. Compared to the first model it is clear that the condition, $V_f = V_c$, hampers the accretion of bodies. Nevertheless, one body eventually grows beyond M_c and incorporates about 20% of the initial total mass. It is also clear that the evolution of mass distribution is expressed by an inverse power relation, $N(m, t) = c\ (t)\ m^{-\alpha}$, with the constant index $\alpha \sim -0.86$. In the fragmentation dominant case, the index α of the inverse power relation is also shown to be constant with time and to be -0.85 ± 0.05 irrespective of initial mass distribution (Matsui, 1977).

To clarify the condition required for early growth of planetesimals, we show

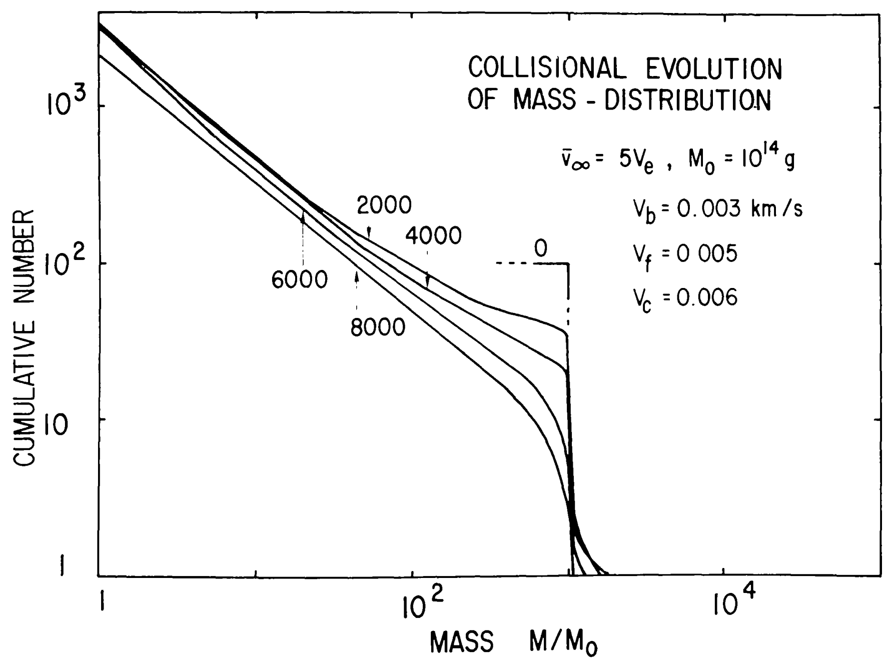

Fig. 3. Collisional evolution of mass-distribution spectrum. Representation is the same as Fig. 2. This model has higher mean relative velocity ($\bar{v}_\infty = 5\ V_e$). The other parameters are the same as the previous model. Finally, all bodies (100%) are fragmented and transferred to the mass range smaller than M_0. Slopes for the smaller mass range are in the narrow range around - 0.82.

the temporal variation of total mass of the bodies which grow beyond M* in Fig. 5. The solid and broken lines represent the models with $V_f = V_c$ and $V_f < V_c$ respectively. It is clear from this figure that low mean relative velocity (at least smaller than 0.005 km/s, about 2 times the escape velocity, V_e, of 10^{17} g body) is a necessary condition for growth beyond M_c. The relation between V_f and V_c also seriously affects the collisional evolution. If $V_f = V_c$, mean relative velocity should be smaller than 0.0025 km/s ($\sim V_e$) for the growth beyond M_c. However, the bodies with the mechanical properties of basaltic rock can grow beyond M_c even if the mean relative velocity is much larger than 0.005 km/s. This is because V_c for basaltic rock is about 10 times larger than that for weaker body and so M_c for basaltic body is about 1.46×10^{21}g. To grow beyond 1.46×10^{21}g similar conditions should be necessary. In Fig. 6 is shown the evolution of the model with mechanical properties of basaltic rocks. This model is characterized as follows: the bodies are initially all of the same mass ($M^* = 10^{20}$g), mechanical properties are similar to basaltic rock ($V_b = 0.03$ km/s, $V_c = 0.063$ km/s, and $V_f = 0.05$ km/s) and $\bar{v}_\infty = 2\ V_e$ where V_e is the escape velocity of the largest body at $t = 0(\sim 0.026$ km/s). It is seen that about 70% of the initial total mass is transferred to the mass range smaller than M_O and the rest (30%) is incorporated by the largest body after 18700 collisions. Compared to the first model the fraction of the total mass of the body with mass larger than M_c ($\sim 1.4 \times 10^{21}$g) is small. However, growth of the largest body beyond M_c also occurs in this model.

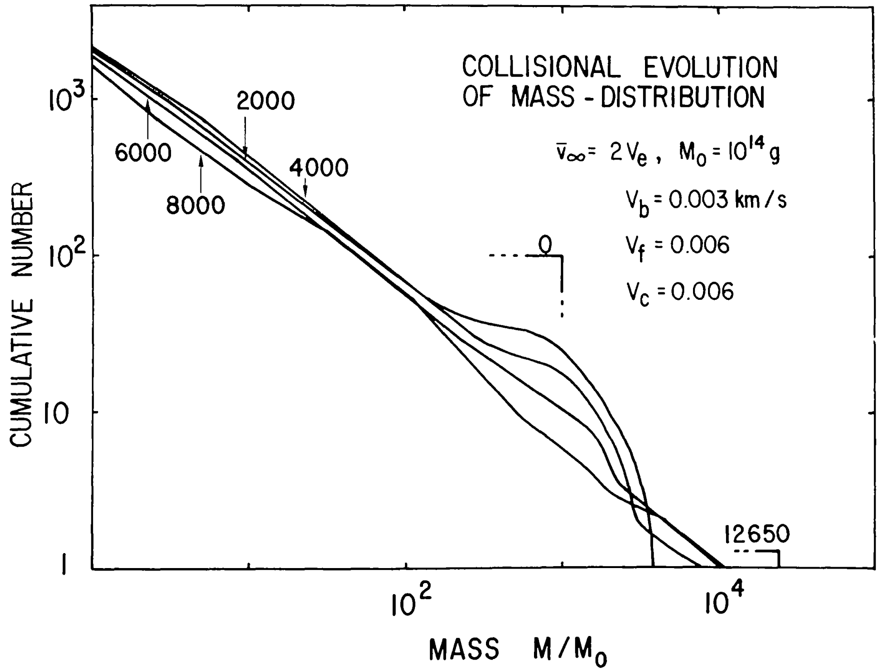

Fig. 4. Collisional evolution of mass-distribution spectrum. Representation is the same as Fig. 2. V_f is equal to V_c in this model. Except for the relation between V_f and V_c the conditions are the same as the first model. Bodies corresponding to about 20% of the initial total mass are transferred to the mass range larger than the initial mass. Slopes for the smaller mass range are in the narrow range around - 0.86.

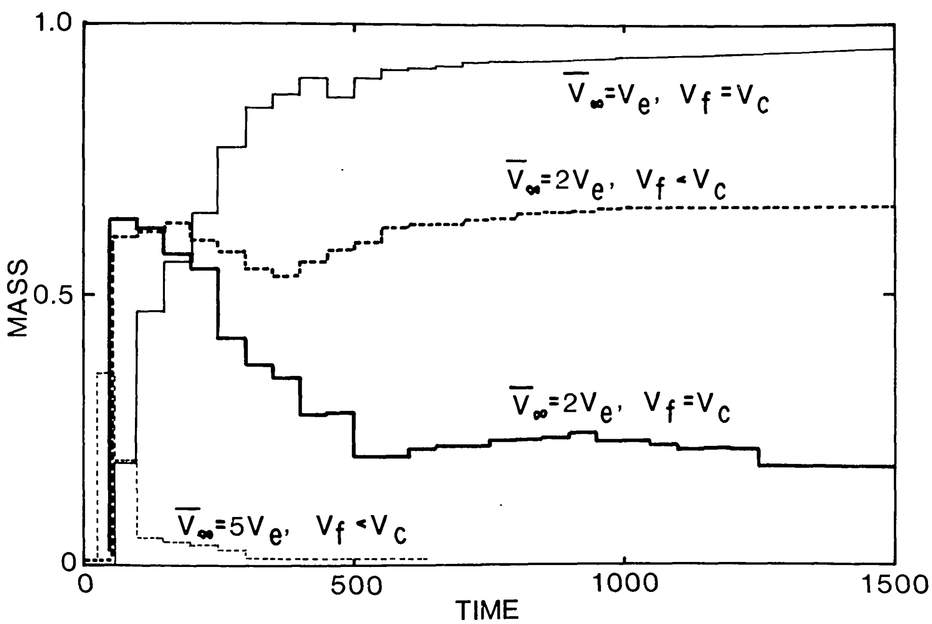

Fig. 5. Temporal variation of total mass of the bodies with mass larger than the initial mass M*. Note that either higher mean relative velocity or $V_f = V_c$ hampers the accretion of bodies. Mass and time are normalized by initial total mass and mean collision interval at t = 0.

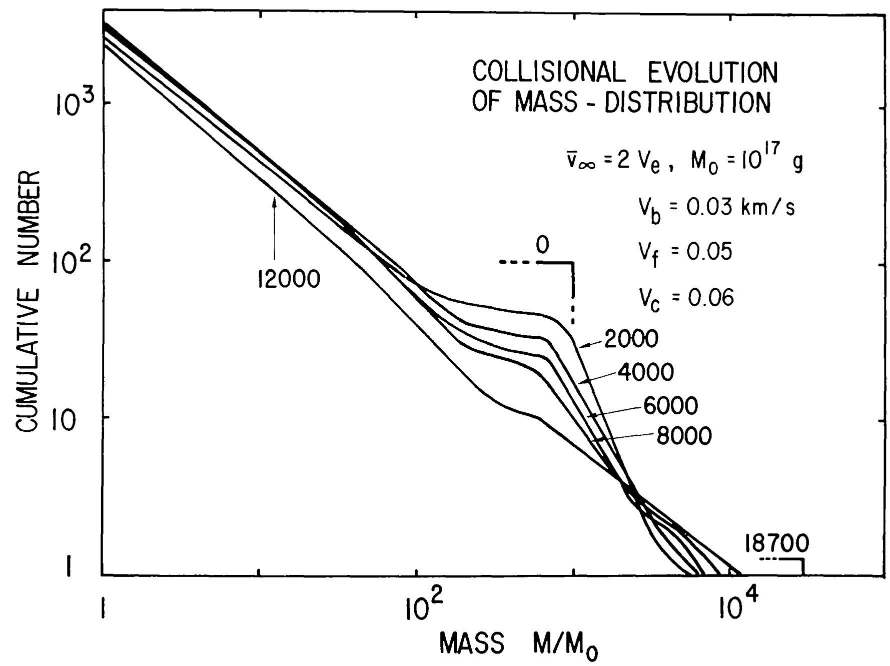

Fig. 6. Collisional evolution of mass-distribution spectrum. Representation is the same as Fig. 2. This model is characterized by the bodies with mechanical properties such as basaltic rock. Bodies corresponding to about 30% of the initial total mass are transferred to the mass range larger than the initial mass. Slopes for the smaller mass range are in the narrow range around - 0.85.

Slope in smaller mass range is similar to that of Figs. 3 and 4, because fragmentation is rather predominant in this model.

DISCUSSION

The Monte Carlo technique adopted in this study is very suitable for those stochastic processes in which the planetesimals collide mutually and change the size distribution with time. Although the initial total number of bodies are very small compared to the real case (in most cases initial total number is from 100 to 1000 in this study), difference in total number is negligible if we treat the growth of bodies within the restricted mass range ($M_0 \leq m < \sim 10^6\ M_0$). The assumption that mean relative velocity is constant with time is also considered to be reasonable in this case because growth time is expected to be faster than the damping time due to inelastic collision.

Derived results are summarized as follows: If mean impact velocity is much smaller than V_c, collision types are pure rebound and coagulation. Therefore, growth rate of the largest body at this stage is dependent on escape velocity, mean impact velocity, and the coefficient of restitution. Particularly the last parameter is important. On the other hand, when mean impact velocity is about V_c or greater than this value, the collision becomes complicated but there occurs mostly either fragmentation or coagulation. The relation between parameters such as $\bar{v}_\infty$, V_c and V_f determine a predominant collision type. Accordingly, the growth rate at this stage depends on escape velocity, mean impact velocity, and velocity distribution of fragments. In any case, evolution of mass distribution is expressed by an inverse power relation such as $N(m, t) = c\,(m, t)\,m^{-\alpha}$ except for larger bodies irrespective of initial mass distribution. Values of α are listed in Table 2.

TABLE 2. Index , α , of Inverse Power Relation, $N(m, t) = c(t)\,m^{-\alpha}$

Predominant Collision Type	Collision Cross Section	α
Coagulation	r^2	0.35 ± 0.05
"	r^3	0.5 ± 0.05
"	r^4	1.0 ± 0.05
Fragmentation	r^2	0.85 ± 0.05
Coagulation & Fragmentation	r^2	0.7 ± 0.05

These results are consistent with the general picture derived by Greenberg *et al.* (1977) and other investigators (e.g. Safronov, 1972); $\alpha = 0.85 \pm 0.05$ for fragmentation case is consistent with the previous works by Dohnanyi (1969), Hellyer (1970, 1971) and Bandermann (1972), $\alpha = 0.5 \pm 0.05$ for coagulation case is consistent with the result derived analytically by Zvyagina and Safronov (1972), and $\alpha = 0.7 \pm 0.05$ is consistent with the numerical solution of generalized coagulation by Pechernikova *et al.* (1976) (their adopted q value is 1.75 and derived α value is 0.66).

Mean impact velocity is one of the most important parameters and so could almost control the collisional evolution of planetesimals. However, very little is known about the temporal variation of mean relative velocity (Safronov, 1972; Greenberg *et al.*, 1977; Matsui and Mizutani, 1978). According to Safronov's estimate, when scattering by close encounter and damping by inelastic collision are balanced, mean relative velocity is expressed by

$$\bar{v}_\infty = V'_e / \sqrt{2\Theta} \tag{6}$$

where V'_e is an escape velocity of the largest body at arbitrary time and Θ is a numerical constant. Results of our numerical simulation show that collisional evolution depends strongly on the value of Θ. If Θ is much larger than ½ as is pointed out by Safronov (1972) and Matsui and Mizutani (1978), coagulation is a predominant mode of collision. Growth rate is dependent on the coefficient of restitution. On the other hand, if Θ is much smaller than ½ ($\sim 1/98$) as is shown by Greenberg *et al.* (1977), fragmentation is a predominant collision type. Collisional evolution is rather complex if Θ is around ½, and the relation between V_c and V_f might play a key role in the collisional evolution for this case. Although it is not our purpose to estimate Θ, we should point out that mean relative velocity might have been at least several km/s sometime during accretion. Because present mean eccentricity of terrestrial planets is ~ 0.08 this leads us to the value of several km/s as the relative velocity (according to Ziglina and Safronov (1976), mean eccentricity for earth region varies with time from 0.0017 to 0.18 by the mutual gravitational perturbations of the growing planet). If this high mean relative velocity can be applied to the early accretion stage, we need to introduce something like nucleating agents (such as iron-meteorite bodies which have plastic ductile properties at temperatures higher than 200°K) for the formation of terrestrial planets (Matsui and Mizutani, 1977). This suggests that planetesimals accrete inhomogeneously due to the difference in mechanical strength: iron being the first, and silicate the second. This is called the *inhomogeneous* accretion model hereafter. The other case is called the *homogeneous* model (that is, the homogeneous model means that planetesimals can grow irrespective of their mechanical strength).

The mass distribution of impacting bodies at the last phase of planetary formation could be very different in homogeneous and inhomogeneous models. For the homogeneous model, many larger bodies can survive a catastrophic type of collision so that the size of impacting bodies is expected to be as large as that of the largest planetesimal. It means that mass ratio between the largest and second largest planetesimal is close to 1. Finally they mutually collide and grow to a full-size planet. Hence the resulting thermal state might depend not only on the time scale of accretion but also on size of impacting bodies (Safronov, 1978). On the other hand relative mass ratio between the largest body and impacting bodies for inhomogeneous model becomes large as the largest body survives catastrophic break-up and grows. [It would have been much larger than 10^3 (Safronov, 1972.)] Rocky bodies mostly experience catastrophic fragmentation so do not gain the mass through their mutual collisions. In this case, the early thermal state of a growing planet is very much dependent on the time scale of accretion, but not at all on the size of impacting bodies.

In summary,

(i) Whether or not the early growth of planetesimals occurs depends mainly on mean impact velocity and mechanical properties as is pointed out by Greenberg *et al.* (1977), Matsui and Mizutani (1977), and Hartmann (1978). If temporal variation of mean impact velocity is expressed by Eq. (6), $\Theta \gg 1/2$ is a necessary condition for the early growth of planetesimals. Growth rate is mostly dependent on the coefficient of restitution in this case. If $\Theta \sim 1/2$, other conditions such as $V_f \ll V_c$ are necessary for the early growth. In this case growth rate is a complicated function of parameters such as mean impact velocity, V_f, k_2 and V'_e.

(ii) Evolution of mass-distribution spectrum is well expressed by an inverse power relation, $N(m, t) = c(t)\, m^{-\alpha}$. Whenccoagulation and fragmentation are competing, α is about 0.7 regardless of initial mass distribution.

(iii) Present mean eccentricity of the terrestrial planets leads us to high mean relative velocities, several km/s. If this high mean relative velocity can be applied to the early accretion stage, we need to introduce something like nucleating agents (such as iron-meteorite) for the early growth of planetesimals. This suggests that planetesimals accrete inhomogeneously due to the difference in mechanical strength.

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