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Research Paper

On the evolution of terrestrial planets: Bi-stability, stochastic effects, and the non-uniqueness of tectonic states

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A R T I C L E I N F O

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A B S T R A C T

The Earth is the only body in the solar system for which significant observational constraints are accessible to such a degree that they can be used to discriminate between competing models of Earth's tectonic evolution. It is a natural tendency to use observations of the Earth to inform more general models of planetary evolution. However, our understating of Earth's evolution is far from complete. In recent years, there has been growing geodynamic and geochemical evidence that suggests that plate tectonics may not have operated on the early Earth, with both the timing of its onset and the length of its activity far from certain. Recently, the potential of tectonic bi-stability (multiple stable, energetically allowed solutions) has been shown to be dynamically viable, both from analytical analysis and through numeric experiments in two and three dimensions. This indicates that multiple tectonic modes may operate on a single planetary body at different times within its temporal evolution. It also allows for the potential that feedback mechanisms between the internal dynamics and surface processes (e.g., surface temperature changes driven by long term climate evolution), acting at different thermal evolution times, can cause terrestrial worlds to alternate between multiple tectonic states over giga-year timescales. The implication within this framework is that terrestrial planets have the potential to migrate through tectonic regimes at similar 'thermal evolution times' (e.g., points were they have a similar bulk mantle temperature and energies), but at very different 'temporal times' (time since planetary formation). It can be further shown that identical planets at similar stages of their evolution may exhibit different tectonic regimes due to random variations. Here, we will discuss constraints on the tectonic evolution of the Earth and present a novel framework of planetary evolution that moves toward probabilistic arguments based on general physical principals, as opposed to particular rheologies, and incorporates the potential of tectonic regime transitions and multiple tectonics states being viable at equivalent physical and chemical conditions.

1. Introduction

Observations of the Earth are often used to inform general models of planetary evolution. It is important to note that the Earth is the only body in the solar system for which significant information about thermal, geologic, and tectonic evolution is currently accessible. However, even with a relatively large dataset, our understanding of Earth's evolution is far from complete. While we know plate tectonics is currently operative, the timing of its onset, the length of its activity, and its initiation mechanism are far from certain (e.g., O'Neill et al., 2007; Debaille et al., 2013; Gerya, 2014). The observation that the Earth is the only solar system body with currently operative plate tectonics can lead to the conclusion that the Earth is unique in terms of its current tectonic state. However, the degree to which Earth is unique in terms of its thermal-tectonic evolution relative to the other terrestrial planets remains an open one. In order to begin to address this question, observations of all main terrestrial bodies, as well as the underlying energetics of mantle convection are fundamentally intertwined, and need to be considered together synergistically.

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A general survey of geologic activity of the three main terrestrial planets in the solar system suggests that the Earth is currently unique in that it operates within a plate tectonic regime. In contrast to the Earth, both Mars and Venus exhibit very different tectonic states. Observations suggest that Mars may operate within what is termed a single plate mode of tectonics, or a stagnant-lid regime (e.g., Nimmo and Stevenson, 2000). In a stagnant-lid regime, the cold and stiff outermost rock layer does not participate in mantle overturn, nor does it exhibit significant horizontal surface motions. In contrast to the stagnant-lid regime, plate tectonics (as manifest on Earth), is characterized by the horizontal motions of strong surface plates. Surface motion is accommodated by localized failure along relatively narrow plate boundary zones. The critical difference between a stagnant-lid and a plate tectonic regime, in terms of a planet's thermal state, is that the cold surface plates of plate tectonics participate in mantle overturn and the associated cooling of the planetary interior. As a result, plate tectonics is considered to be an example of a mobile-lid style of mantle convection (also referred to as active-lid convection). In contrast to observations of both Earth and Mars, it has been suggested that Venus has been, and perhaps still is, operating in an episodic-lid regime (e.g., Turcotte, 1993; Fowler and O'Brien, 1996; Moresi and Solomatov, 1998). The episodic regime is highly dynamic, characterized by periods of extreme quiescence (akin to stagnant-lid) punctuated with rapid episodes of surface overturn and mobility (Moresi and Solomatov, 1998).

While the nature of tectonics that the early Earth exhibited is hotly debated (e.g., Davies, 1993; Calvert et al., 1995; O'Neill et al., 2007, 2013, 2016; Condie and Kroner, 2008; Stern, 2008; Moyen and van Hunen, 2012; Debaille et al., 2013; Foley and Bercovici, 2014; Gerya, 2014; O'Neill and Debaille, 2014; Weller et al., 2015), an important aspect in planetary evolution that has long been in consensus is that as the Earth cools the driving energy for plate-tectonics will wane, and the Earth will begin to move into stagnant-lid regime similar to observations for current day Mars (e.g., Nimmo and Stevenson, 2000). The implication is that the lid-state of a planet can change over time. Recently, this idea has been bolstered through several studies exploring the sensitivity of mantle convection and lid-states to changes in internal temperature, through internal heating and/or long term climatic effects (O'Neill et al., 2007, 2016; Lenardic et al., 2008, 2016a; Landuyt and Bercovici, 2009; Foley et al., 2012; Lenardic and Crowley, 2012; Stein et al., 2013; Weller et al., 2015; Weller and Lenardic, 2016; Weller et al., 2016).

The different tectonic states of terrestrial planets in our solar system allow for two end-member viewpoints regarding the nature of planetary tectonics. The first is that differing tectonic expressions can be tied back to an initial difference in physical and/or chemical properties between the planets. For example, planet X starts with two ocean masses of water, while planet Y starts with 10^{-3} ocean masses, or planet X has a lithospheric strength greater than that of planet Y due to variable chemical compositions. This has been the prevalent view of planetary systems, parameter α leads to lid-state β mapped into a 1:1 functional relationship. The most dominant thinking along this line is the idea that it is water that allows for plate tectonics on Earth and the lack of water on Venus and Mars leads to different tectonics. A complete list of references are well beyond the scope of this paper, and indeed the reader's attention, but a few early examples that serve to form the basis follow – Tozer, 1985; Mian and Tozer, 1990 – with earlier work arguing water is required to weaken rocks/subduction zone sufficiently to allow for movement – Hubbert and Rubey, 1959; Bird, 1978 – which is an inherent assumption within models that allow for weak (mobile) lithospheres (e.g., Moresi and Solomatov, 1998). It is important to state that this argument is based on a sample size of one planet with active plate tectonics, and as such, it remains an open and intriguing question. If the tectonics of each planet is predominantly the result of such unique differences, then using data obtained from one planetary source to inform models of planetary evolution for another planet may not be strictly applicable. They are simply not equivalent or even nearly equivalent systems from the start (an apples to kumquat analogy), and inferences from apparent similarities could by highly misleading. The second end-member view is that historical contingency is a more critical control on the expression of tectonics, as opposed to the value of a specific parameter, or parameters (Lenardic and Crowley, 2012; Weller and Lenardic, 2012; Weller et al., 2015; Lenardic et al., 2016a; O'Neill et al., 2016). The intent of this paper is not to argue for the uniqueness, or non-uniqueness of the Earth, or any terrestrial planet, but to instead develop the second end-member view above and to offer an alternative framework of evaluating the evolution of terrestrial planets collectively in terms of general evolutionary features that may apply, and be applied to all of them, albeit at different evolution times (e.g., the idea that all terrestrial planets are likely to have transitioned between different tectonic modes over time – as was suggested by Sleep, 2000). This idea, based on recent interpretations of historical data from the Earth, in conjunction with physical and energetic arguments, seeks to place the terrestrial planets into a broad framework of tectonic evolution that can highlight were similarities are likely and were significant divergences are to be expected (we stress that tectonic evolution is distinctly different from current tectonic state).

2. A case study of fluctuations: a record for Earth's tectonic evolution

Critical to the ideas of this paper, and indeed those of thermal and lid-state evolution, is the hypothesis that the tectonic state of a planet can change over its geologically active lifetime. With this in mind, it is important to (briefly) review the growing evidence that suggests planetary tectonic regimes do transition. Given the Earth has the most robust record, it is useful to focus this discussion on the Earth system initially, and expand later to the other terrestrial planets.

Recently several studies based both on geochemical and geodynamic methods have argued for stagnant to episodic behavior in the Archean through the Precambrian (summarized in Fig. 1) (e.g., Debaille et al., 2013; O'Neill et al., 2013; O'Neill and Debaille, 2014). Individually, the records do not, and cannot, provide definitive evidence for variable lid-states. This in turn has led to an ongoing debate that revolves around whether this episodic record is a primary feature of the Earth system, or is the result of incomplete preservation related to a highly dynamic system (e.g. destruction and overprinting from supercontinent aggregation and dispersal) (Cawood et al., 2013). At a fundamental level, this argument is one of time bias. However, taken in aggregate, these several disparate sources provide reasonably compelling evidence for significant changes in the linked internal/external processes of the Earth over time.

Probably the most direct line of physical evidence for surface motions (or plate activity) in the past is that of paleomagnetic apparent polar wander (APW) paths, which indicate the motion of the plate with respect to a 'fixed' geocentric axial dipole (apparent plate velocities: Fig. 1). However, specific identification of plate motions in the past is difficult due to often high uncertainty inherent in paleo-pole positions, as well as large temporal sampling biases, some of which may span 100s Myrs due to the sporadic record. These uncertainties can make the construction of coherent apparent-polar wander paths particularly problematic. However, in the last decade, high-quality apparent polar wander paths have

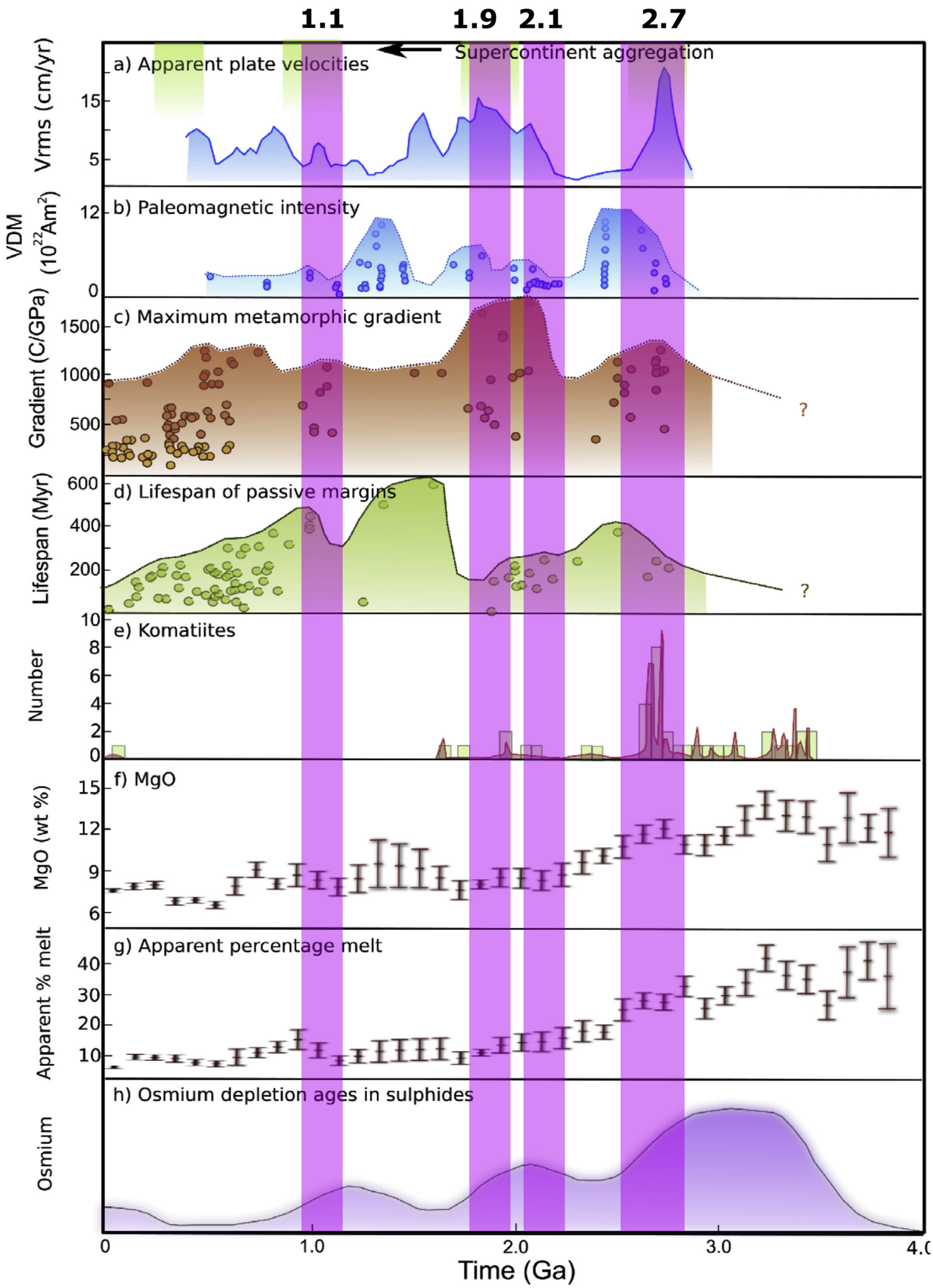


Figure 1. (A) Root-mean square apparent plate velocities, based on paleomagnetic data, compiled by Piper (2013) with inferred times of supercontinent assembly from Condie (2004). (B) Paleomagnetic intensity (expressed as virtual dipole moment), from the compilations of Macouin et al. (2004) and Reddy and Evans (2009). (C) Maximum metamorphic gradients, from the compilation of Brown (2007, 2014). (D) Lifespan of passive margins, from Bradley (2011). (E) Distribution of komatiitic volcanism through time, from Isley and Abbott (1999). (F) MgO content of mafic melts through time, from Keller and Schoene (2012). (G) Apparent melt percentage of the mantle source, from Keller and Schoene (2012). (H) Osmium depletion ages for cratonic sulphides, from Pearson et al. (2007). Purple lines indicate proposed episodic events of O'Neill et al. (2007). Figure modified after Lenardic et al. (2016b).

been constructed for the Precambrian using stable cratons (e.g., high information density datasets) which showed apparent wander velocities ranging from well over 100 cm/yr to essentially zero, a change of O(10–100) in apparent plate velocities for set periods of time (O'Neill et al., 2007). The variation of extreme activity, indicated by purple shading in Fig. 1 at 2.7, 2.1, 1.9, and 1.1 Ga, to periods of relative quiescence, indicated in spaces between purple shading in Fig. 1 from 2.7 to 2.1, 2.1 to 1.9, and 1.9 to 1.1 Ga (otherwise known as the 'Boring Billion') was argued to be indicative of an episodic mode of mantle convection. With the advent of higher quality data products, the consistency of the O'Neill et al. (2007) assertion was tested, and the Precambrian paleo-poles were found to overlap for significant intervals of time, which is further consistent with a stagnant-lid, or near stagnant-lid, mode of tectonics that is interspersed with large amplitude apparent polar wander events (e.g., episodic motions). In short, periods of episodic activity of the surface is inferred by apparent plate motions that increase substantially (of O(10), purple shading Fig. 1A) from a more general, low to negligible velocity quiescent background rate. Related to the apparent surface motions, variations in the intensity of the paleomagnetic field (Macouin et al., 2004; Reddy and Evans, 2009) followed the APW peaks after an offset interval of ~50–100 Myrs (O'Neill et al., 2013). This physically is consistent with the transition time of subducted/overturned surface material to the core mantle boundary (CMB). The positioning of cold material along the hotter CMB boosts heatflux from the boundary layer, and as a net result, increases the magnetic field strength over short intervals of time. In aggregate, these observations suggest there were fluctuations in the tectonic regime throughout the Precambrian.

With the extreme fluctuations indicated by estimations of plate velocities in the past, the natural implication would be a distinctive tectonic signature preserved in the geologic record. The issue with any unambiguous identification of just such a signature is the overlap and cyclicity of supercontinents (Cawood et al., 2013). However, while the supercontinent cycle itself may obscure, or destroy prior evidence in the record, it too may also be indirect evidence. Supercontinent assembly is significantly more likely when plates are moving, and moving rapidly (green shading Fig. 1, Condie et al., 2009). Further, distinct orogenic and dispersal signatures are in evidence at times of identified paleomagnetic excursions (e.g., Condie et al., 2009; Fig. 1). An interesting, and highly suggestive metamorphic petrology dataset suggests that a signature of a modern style of plate tectonics, characterized by paired lower T/P in the subduction zone and higher T/P in the overriding plate, is not established until the Neoproterozoic (Brown, 2007, 2014). These geothermal gradients often indicate peaks at specific intervals, often associated with supercontinent aggregation, but can also be associated with plate velocity fluctuations. These gradients may reach extremes of ~2000 K/GPa at ca. 1.9 Ga (Brown, 2007). Interestingly, the passive margin lifetime also peaks near 2.5 Ga and 1.6–1.4 Ga (Bradley, 2011). It would be expected that the longevity of these passive margins to be anti-correlated with tectonic activity, and further supports the relative quiescence of these margins post orogenic events (Fig. 1).

Another significant line of evidence for tectonic regime changes may be reflected in the volcanic record. Within either a plate tectonic or episodic regime, volcanism may be predicted to largely follow with variations in tectonic activity. It has been known that there is a distinct time-dependence to both the large-igneous province (LIP) record, and komatiite distribution (Isley and Abbott, 1999). This in conjunction with mantle depletion curves, recorded from either Nb/Th ratios in melts (Condie, 2004) or osmium model ages (Pearson et al., 2007), suggests the Precambrian Earth was replete with punctuated, and large scale volcanic events. Interestingly, records of the evolution of mantle temperatures over time show marked inflections near ~3 Ga (Herzberg et al., 2010). Recently, Keller and Schoene (2012) statistically estimated apparent melt percentages to average between 30% and 40% in the Archean, while decreasing significantly to less than 20% in the Proterozoic. Within that broadly decreasing trend is a time dependent fluctuation indicating melting spikes at ~3.2 Ga and 2.7 Ga, which are largely echoed in the MgO content over the same period of time.

In summary, observations in the Earth record of apparent plate velocities, paleomagnetic intensities, metamorphic gradients, passive margin lifetimes, and geochemical signatures (to name a few) strongly suggest periods of extreme activity (at 2.7, 2.1, 1.9, and 1.1 Ga), and periods of relative quiescence (2.7–2.1, 2.1–1.9, and 1.9–1.1 Ga). These data are consistent with definition of an episodic-lid mode of convection. This then suggest that the Earth's middle period may have in fact been highly dynamic, with strong regional/hemispheric variations in tectonics and observables, and that the 'Boring Billion' may have been anything but boring.

3. Physics and numerical models of lid-transitions over time: effects of internal heating on convection

We now have multiple lines of significant, if indirect, evidence for fluctuations in the tectonics of a case study planet, the Earth. The data indicates a relatively quiescent early Earth (post late heavy bombardment), punctuated by periods of extreme activity (from ~3 Ga to ~1 Ga), with perhaps a modern style of plate tectonics emerging within roughly the last Gyr. The key question then becomes - is this temporal pathway of evolution (stagnant/near stagnant to episodic to mobile) representative of general planetary evolution? To begin to address this, we now turn to numerical experiments for which critical physical factors affecting system evolution, and inherent system feedback effects, can be isolated and explored.

Recently, Weller et al. (2015) and O'Neill et al. (2016) showed that the thermal evolution of planetary convective systems with high levels of internal heating strongly favor early (hot) stagnant-lid states. However, as radiogenic heating is tapped, the hot stagnant-lid can yield through an intermediary episodic-lid, into a mobile-lid regime. With a further decrease in radiogenic heating, the mobile-lid transitions back into a (now) cold stagnant-lid (summarized in Fig. 2, and the reader is referred to Weller et al. (2015) and O'Neill et al. (2016) for model specifics).

Here it is worth briefly (re)outlining how regimes are defined. These definitions and metrics will be used throughout the remainder of the paper. Mobile-lids are identified by active yielding of the surface, with appreciable horizontal motions and interaction with the deep interior (e.g., subducting slabs). Further, in these cases the surface velocity is near that of the interior velocity (or within the range of ~0.8–1.8 the internal velocity). Stagnant-lids by contrast show highly limited (e.g., 'resurfacing' times greater than the planet's lifetime) to no surface motions, with no active yielding or communication with the interior. Surface velocities in these cases will be far less than internal velocities (<0.1, often <0.01, internal velocities). For the same parameter values, stagnant-lids have thicker boundary layers, lower heatflux, and higher internal temperatures. Both results are defined from statistically steady state conditions. Episodic regimes oscillate strongly between both end-member states, with surface velocities of an O(10) increase from mobile-lid values.

Of interest is understanding the fundamental physics that allows for planetary regime changes with changing internal heating rates. The cold, or late-stage, stagnant-lid end-member is associated with low levels of internal heating. As a result, Weller et al. (2015) described it as being in a low overall state of convective vigor, such that the internal velocity of convection is low, and as a result,

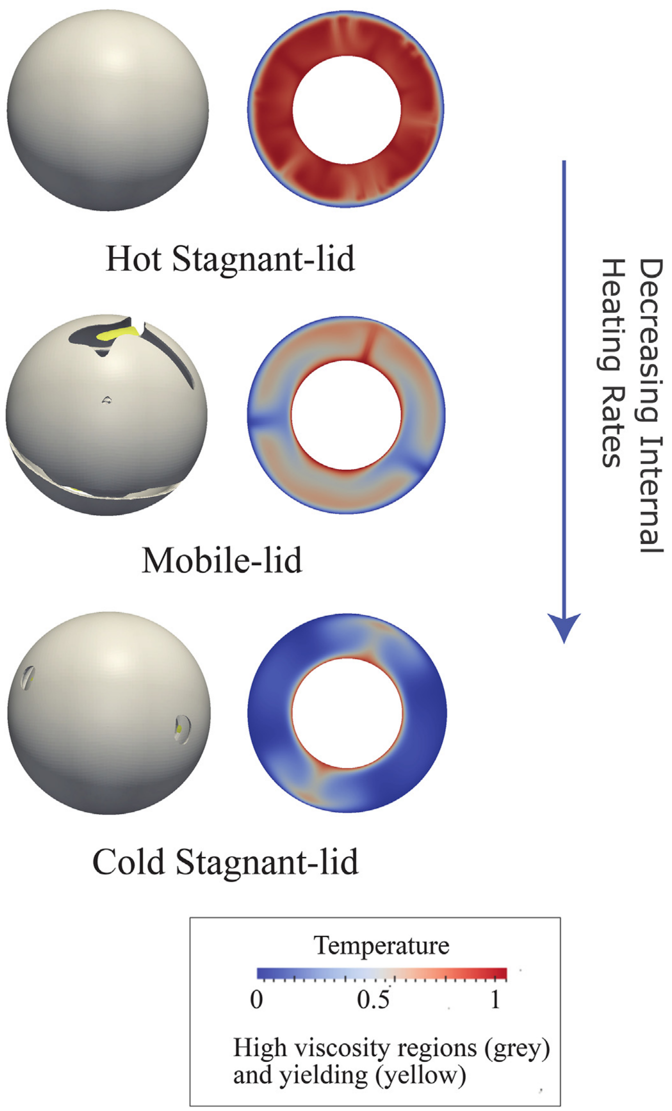


Figure 2. Effects of internal heating on tectonic regimes. Internal heating is varied from high to none. Shown are viscosity plots (left– grey shells: high viscosity "plates"; yellow bands: regions of yielding) and non-dimensional thermal profiles (right) from the Core Mantle Boundary (CMB) to surface. The system exists as a hot stagnant-lid for high internal heating rates, a mobile-lid for intermediate internal heating rates, and in a cold stagnant-lid for low internal heating rates (modified after Weller et al., 2015).

the convective stresses imparted to the lithosphere drop below the stress required to mobilize it. In contrast, the hot stagnant-lid is associated with high degrees of convective vigor and, as a result, potentially large velocities. The above argument, and indeed canonical models of convection would imply that the systems then should be in a mobile state. Weller et al. (2015) suggested that the reason for this stagnant-lid state was rheological, and that the convective stress could still be low due to feedback in the temperature-dependent viscosity of a high internal temperature mantle. The argument follows that high temperatures lead to a low internal mantle viscosity, which has the effect of lowering convective stresses imparted to the lid through a decoupling of the mantle from the lithosphere (e.g., a process in principle similar to that suggested for Venus by Buck (1992)). While mantle rheologies and specific lithosphere strengths should certainly have an effect on the coupled lid-stress state of a planet, this explanation by itself is not entirely satisfactory. Specific rheologies used in models depend on poorly known material parameters (for the Earth let alone any other planet) that may not be universal (i.e., dynamic behavior that is connected to a constitutive equation or material property may be specific to the material property itself, as opposed to being tied to the more general energetics of the system). A more fundamental approach, that has the potential for broader applicability, would be one that is based on the energetics of convection independent of any specific rheological flow law and/or lithospheric strength. Here specific refers to just that, a specific rheologic law with the understanding that any rheology is subject to the fundamental constraints inherent within energy conservation.

Weller et al. (2016) and Weller and Lenardic (2016) have recently explored the energetics of mixed heating mode convecting systems with variable degrees of internal heating rates. One of the key findings of these studies was that convective velocities, a measure of the energy available to do work in a convective system, systematically decrease and plateau with the application of greater rates of internal heating (summarized in Fig. 3). Results are plotted as a function of a boundary layer *Ra* (Ra_b see figure text) in Fig. 3A. For a given thermal *Ra* (Ra_t), defined at the base of the mantle and using the temperature contrast from the CMB to surface, increasing heat production shifts values to the left of the plot (first point on the left of a given Ra_t is basally heated, last point on the right is maximal internal heating, following Q_{sc} in Fig. 3B). These results stand in stark contrast to the standard assumption that is often applied to convective systems, that of greater velocities (and ability to do work) for younger planets with higher levels of internal heat and convective vigor. It was argued that this effect was predominantly that of a dependence of aspect ratio on internal quantities, and as a result, a decrease in velocity with increasing internal heating rates was to be qualitatively expected, as too were maximum velocities for cases of no internal heating. For isoviscous systems, Schubert and Anderson (1985) showed that an increase of heat from internal heating rates tended to move a system toward smaller convective cell aspect ratios, which are associated with lower overall average velocities. Additionally, as heat is applied to the system, the temperature drop from the upper to the lower mantle increases due to the development of a subadiabatic thermal profile, which results in a stable density layering of the mantle, that can further decrease convective vigor. The net results suggested that the ability of a convective system to impart stress (ability to do work) on the lid is a function of the level of internal heating, or taken as a proxy, a function of time. These results offer a simple physical framework for the propensity of (1) the existence of a hot, early, low-stress stagnant-lid, and (2) the ability of this state to transition to another tectonic state as the global convective cell aspect ratio, and likely also the coupling between the lithosphere and the convecting mantle increases. The implication, then, is that mobile-lids can arise naturally through the reduction in internal heating rates without the need of external effects, or special parameters (such as water), from feedbacks between stress, aspect ratio development, convective velocities, and lid-state coupling. The implication, then, is that the feedback that enables mobile-lids initially, when allowed to progress with time, begin to close the window on the mobile-lid epoch of planetary evolution, resulting in a planet that operates in a later stage, cold, stagnant-lid (Weller et al., 2015; O'Neill et al., 2016).

4. Bi-stability: temporal evolution with history dependence

In contrast to the lid-state pathway discussed in the previous section, i.e., that of decreasing radiogenic heating, it has been shown that increasing surface temperatures, operating on geologic time scales can cause a transition from mobile-lid convection, into and through an episodic-lid regime, into stagnant-lid behavior (Lenardic et al., 2008; Landuyt and Bercovici, 2009; Foley et al.,

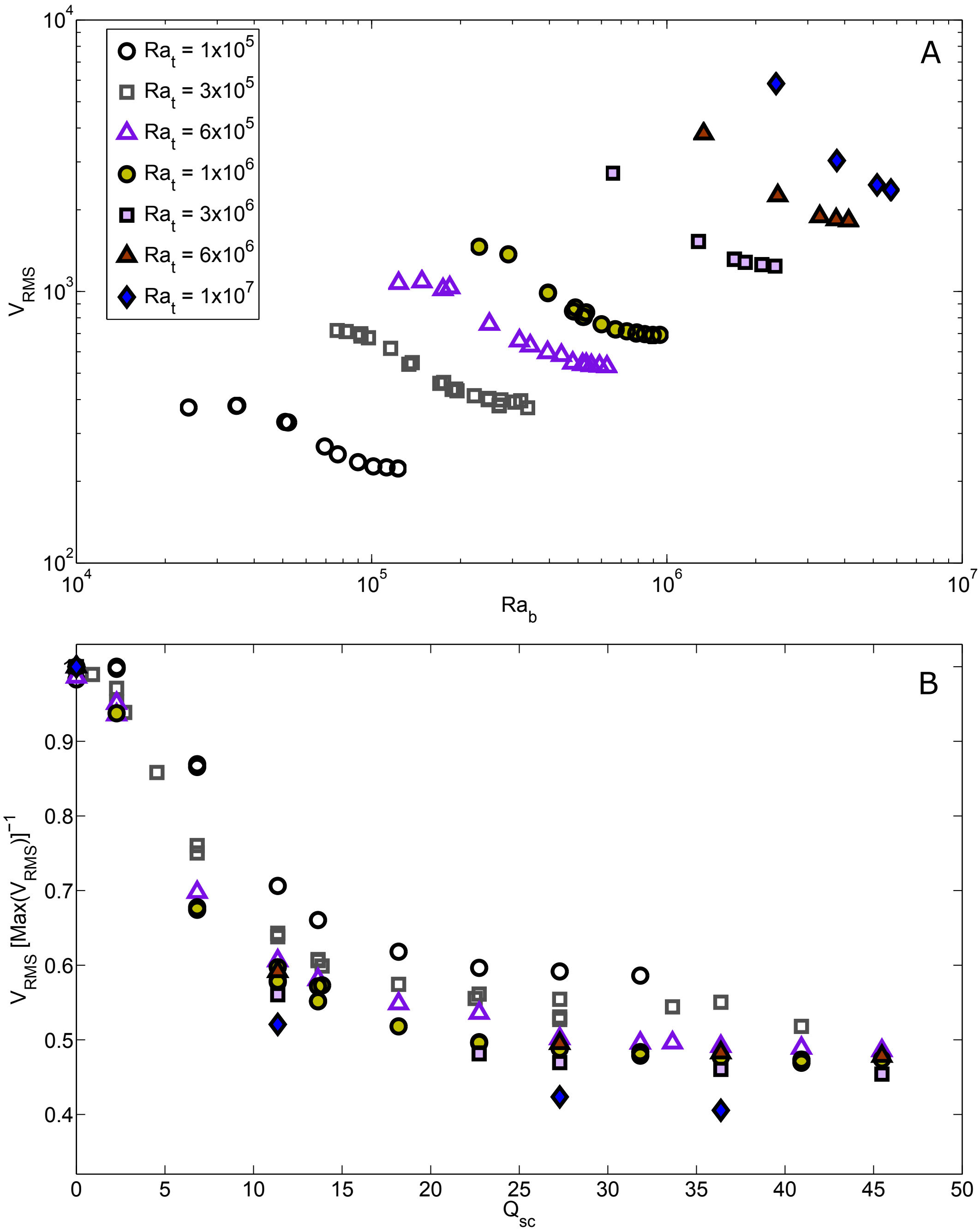


Figure 3. (A) Internal root mean square velocities ($V_{\rm rms}$) vs. boundary layer Rayleigh number ($Ra_{\rm b} = Ra_{\rm t}\Delta T_b$, where ΔT_b is the temperature drop across the surface boundary layer; $Ra_{\rm t} = g\rho\alpha\Delta T d^3/(\kappa\eta_0)$, α is the thermal expansivity, ρ is density, g is gravity, κ is the thermal diffusivity and d is layer depth. ΔT is the reference temperature drop taken from the surface to the CMB. The reference viscosity η_0 is that of the surface). A range of internal heating rates ($Q_{\rm sc}$: 0–45) are plotted for each thermal Rayleigh number ($Ra_{\rm t}$). (B) Normalized $V_{\rm rms}$ vs. Q. Q values shown are scaled from CitcomS input Q to an effective scale Q following $Q_{\rm sc} = \frac{(1-f^3)}{3}\frac{(1-f)}{f}Q$, where f is the core ratio of 0.55 (Weller et al., 2016; Weller and Lenardic, 2016).

2012; Weller et al., 2015). These results indicate that complex feedback processes exist between tectonic state, radiogenic heating (temperature/time), and atmospheric evolution.

Complex feedback effects in an otherwise deterministic system (e.g., mantle convection) at a fundamental level are associated with inherent non-linearity. This can hamper the ability to make predictions regarding the system state at any specific time. It can also allow multiple solutions to exist for the same control parameter (or set of parameters). While the presence of non-linear terms in the equations that are at the heart of mantle convection models is acknowledged, it was often thought that planetary convecting systems were not affected by non unique solutions (e.g., Turcotte and Schubert, 2005), or if noticed that such behavior would be limited to a highly restricted portion of parameter space, and as a result was not followed up on (e.g., Tackley, 1998, 2000). Over the last five years this idea has been challenged, with regions of bi-stability (multiple stable solutions) having been shown to be increasingly important, both analytically (Crowley and O'Connell, 2012), and through numeric experiments in two and three dimensions (Lenardic and Crowley, 2012; Weller and Lenardic, 2012; Weller et al., 2015; Lenardic et al., 2016a; O'Neill et al., 2016). At a fundamental level, bi-stability arguments suggest that no single tectonic regime may be preferred for a given set of parameters. Another way to state this is that within bi-stable regions, stagnant-, episodic-, or mobile-lid states are equally allowable. This indicates that the mode of tectonics the system expresses at any given absolute time (i.e., planetary age from initial formation time) is a strong function of its particular evolutionary pathway.

The potential of bi-stable behavior is already hinted at by the existence of tectonic regime transitions but can go unacknowledged if we only consider the evolution of a single planet. If we only have observations of one planet, e.g., Earth, that transitions from one tectonic regime to another, the assumption can be that beyond the transition point, which is a bifurcation point, there is only one stable regime that the system can move to — it's all we will observe after all in the geologic record. However,

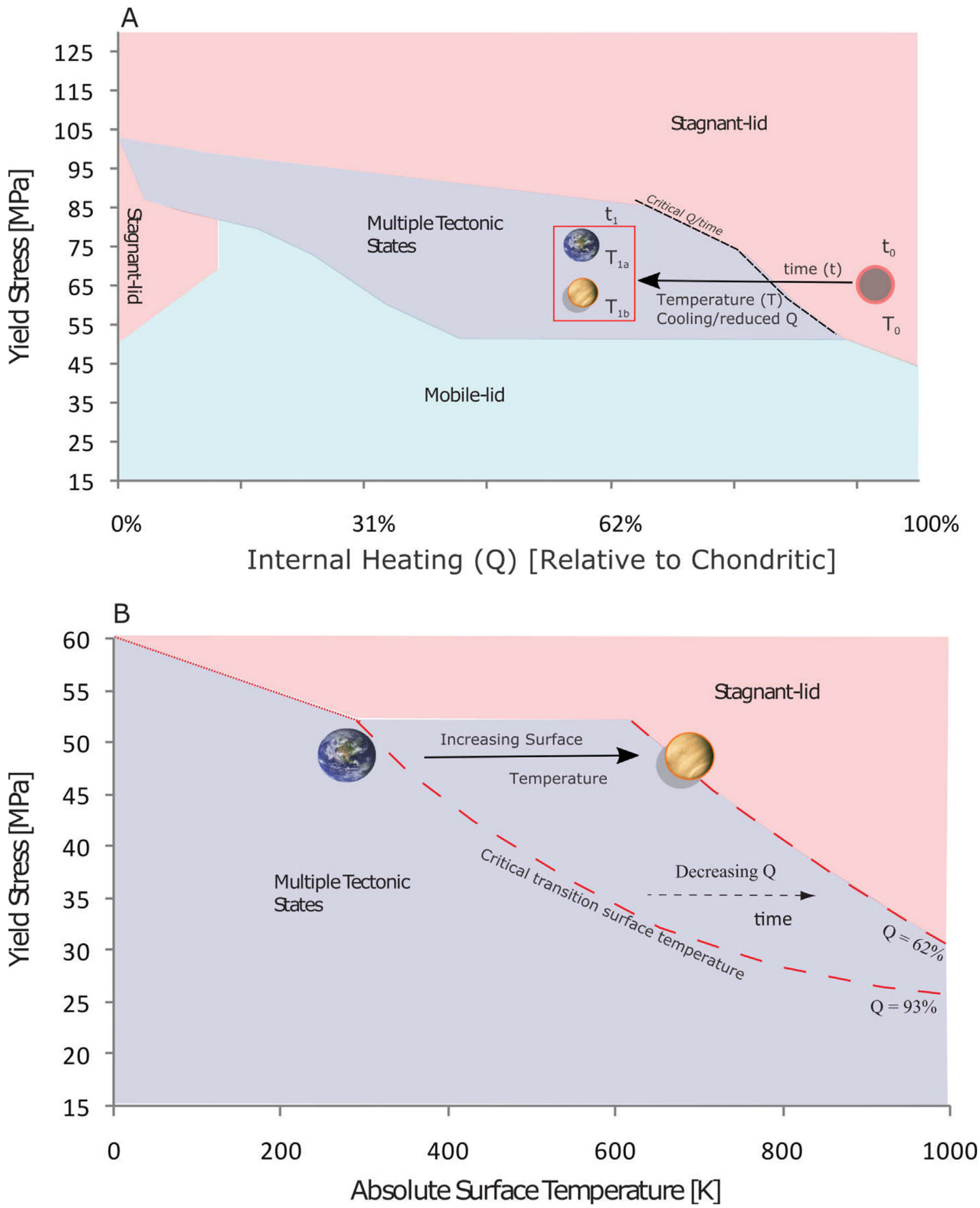
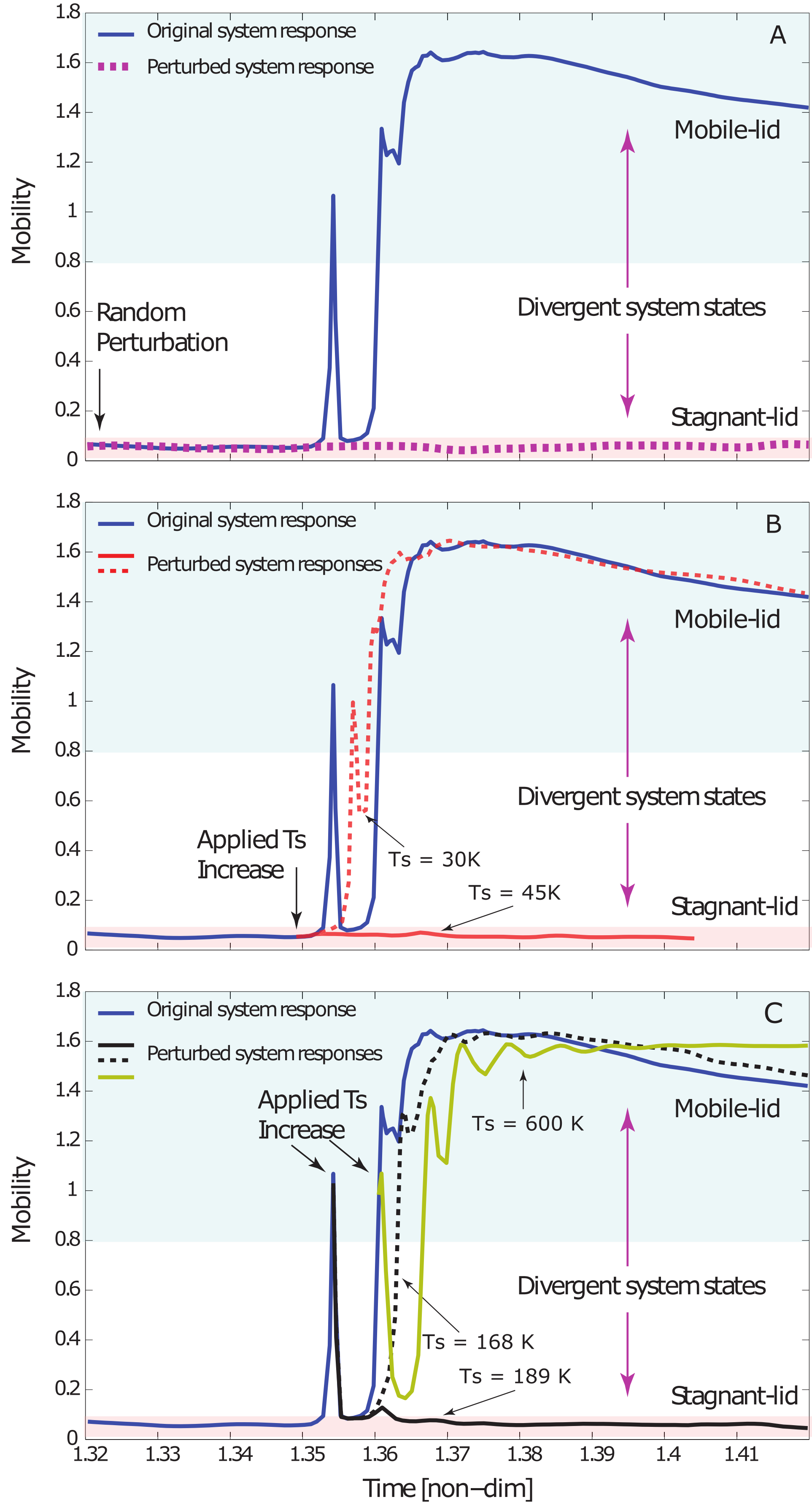


Figure 4. Diagrams showing regions of possible multiple stable tectonic regimes for a planet plotted for, (A) yield stress and internal heating rates parameter space (scaled to values expected for chondritic assumptions of long-lived heat producing elements); and (B) yield stress and absolute surface temperature parameter space in the mobile-lid regime of the internal heating parameter space. Decreasing Q values are shown for the $Q = 93\%$ and $Q = 62\%$ cases to illustrate the widening of bi-stability space as Q decreases and larger surface temperature changes are required to initiate transitions into stagnant-lids (dashed critical transition surface temperature lines). The dashed critical Q/time line in Fig. 4A indicates the yield/internal heating (time) combination required to transition out of an early/initial stagnant-lid state for a planet (denoted by t_0). Internal heating relative to chondritic is a proxy for time (or age), such that at the onset of solid state convection no radiogenic depletion has occurred (full heating; 100%). A point plotted at 62% indicates 38% of radiogenic heat sources have been tapped, while a point at 0% indicates all radiogenic heating is depleted (system becomes basally driven). Sample planets (Earth and Venus) are plotted in multiple state space for illustrative purposes as the current, as well as the time evolution of the yield strength is poorly known. All things being held equal (e.g., yield strength), within this state, either planetary case is allowable from the same initial condition at temperature (T) and time (t_0). These separate cases are denoted by different 'thermal times' (see text for definition) at $T_{(1a,\,1b)}$ at the same absolute time t_1 indicated by the red box. Here Venus and Earth are plotted separately to illustrate multiple states are possible from the same initial condition, that is not to imply that we know one yield strength is greater/lesser than the other. In Fig. 4B Venus and Earth are placed according to their absolute surface temperature. Yield position is chosen to remain self similar between the two cases, and should again be considered illustrative only (modified after Weller et al., 2015).


Original system response
Perturbed system response
A
Mobile-lid
Random
Perturbation
Divergent system states
Stagnant-lid
Mobility
Original system response
Perturbed system responses
B
Mobile-lid
Divergent system states
Applied Ts
Increase
Ts = 30K
Ts = 45K
Stagnant-lid
Mobility
Original system response
Perturbed system responses
C
Applied Ts
Increase
Ts = 600 K
Mobile-lid
Divergent system states
Ts = 168 K
Ts = 189 K
Stagnant-lid
Mobility
Time [non–dim]

that singular evolutionary observation cannot rule out that multiple stable states can in fact exist beyond a bifurcation point, leading to an observational bias. If this alternate reading of the record is allowed for then hysteresis effects inherently enter into discussions of tectonic regime transitions. Hysteresis effects have long been acknowledged in the rock mechanics communities. For example, it is well known that intact rock samples are much stronger than damaged, failed, or fractured samples. Therefore it's easier to deform a failed sample than an intact one. On some level this has been understood in mantle convection (e.g., it's easier to continue plate tectonics than it is to start it). However on another level, until recently, the implications have not been fully acknowledged. Inherent in the assumption of singular tectonic states for any given set of control parameters is the idea of 'reversible', or 'recoverable' tectonic regimes (an "elastic" tectonic shift). That is to say the system response is always weakly nonlinear. Phrased another way, an increase in parameter α to $\alpha+1$ always leads to lid-state $\beta+1$, and a decrease of $\alpha+1$ to α always return to lid-state β (where α is a generic parameter that could be as simple as internal heating rates, or as complex as hydration and dehydration effects on global lithospheric yield strength). Bi-stable behavior and an associated hysteresis introduce a stronger nonlinear mode of behavior and the one to one correspondence between control parameters and tectonic state breaks down. As a result, a return to the original parameter condition may be insufficient to return the system to its original regime as the system may no longer be operationally the same as it was before (i.e., internal temperatures will vary between lid-states).

To illustrate this effect, Weller and Lenardic (2012) ran statistically steady state numerical experiments to show that the yield strength required to transition from a stagnant to mobile-lid was less than the yield strength required to transition from mobile to stagnant. The difference between the two values was termed the Tectono-Convective Transition Window (TCTW), or bi-stable region. The TCTW was found to increase as a power law with increasing vigor of convection (Ra_t), or increasing degrees of temperature-dependent viscosity. Both Ra_t and the temperature dependent viscosity contrast in a planetary mantle are expected to increase for larger terrestrial planets, or increasingly energetic planets such as the early Earth and Venus (in terms of Ra_t).

While there may be a case for yield strengths changing with time, it is more useful to recast bi-stability in terms of a time proxy, that of radiogenic heating. A large suite of three dimensional spherical experiments was performed in order to map out a regime diagram (Fig. 4A) of bi-stable behavior. While the internal heating rate has been scaled to chondritic values, the absolute yield strength values should be considered qualitatively (e.g., high to low). The regime diagram is based on a simple numerically tractable convective system with a fixed lower range value for Ra_t (e.g., $Ra_t = 1 \times 10^5$, viscosity contrast $= 1 \times 10^4$, and variable internal heating rate scaled to chondritic values; see Weller et al., 2015 for model specifics). While the absolute numerical values would be expected to change with increasing vigor of convection (an increase), the trends and relative behaviors are expected to remain robust. In this system, the region of bi-stable behavior occurs for intermediate yield strengths ($\sim$50 MPa $< \sigma_y <$ $\sim$105 MPa) over variable internal heating rates (or time). Mono-tectonic states exist for exceptionally low (mobile-lid $<$ 45 MPa) or high yield strengths (stagnant-lid $>$ 105 MPa). The 'height' of the bi-stability window is indicated to scale as $\sim Ra_t^{2/3}$, indicating a factor of 6 increase in the Ra_t results in a factor of $\sim$3.3 increase in the upper yield strength limit of the bi-stable regime (e.g., maximum mobile-lid yield strength increases from 105 MPa to 346 MPa for an increase in Ra_t and viscosity contrast to 6×10^5 and 6×10^4, respectively). These results are in agreement with the two dimensional cases of Weller and Lenardic (2012).

Taking into account atmospheric evolution (in this case surface temperature) expands the prevalence of bi-stability. Fig. 4A does not consider changes in, or the effects of surface temperature (non-dimensional T_s is 0; see Weller et al. (2015) for further details). The mobile-lid subset of the internal heating parameter space from Fig. 4A is indicated in Fig. 4B (e.g., $\sigma_y <$ 50 MPa). These conditions allow for plate tectonic like behavior for high degrees of internal heating. Due to its high internal temperature, this particular state is also sensitive to changes in surface temperature. Long-lived surface temperature changes in the range of 1–5% of the CMB temperature (here taken as 3287 K in model space; e.g., the CMB temperature with the adiabat removed) have the potential to cause an early mode of plate tectonics to become unstable. As internal heating rates (or temperatures) decrease, the system becomes increasingly insensitive to surface temperatures changes (indicated by dashed lines Fig. 4B). For a planet with $\sim$38% of its original internal heating rate a significant increase in the surface temperature, of up to 30% of the CMB value, no longer induces a tectonic transition (Weller et al., 2015). That is the critical transition surface temperature line in Fig. 4B vanishes as a planet cools. Additionally, Weller et al. (2015) showed increasing surface temperatures in excess of $\sim$1000 K, results in a system that is unable to form stable plates, and as such, classic regime definitions no longer apply (neglecting melt). In the parameter space of Fig. 4B, multiple regimes exist for both lower yields ($\sigma_y \leq$ 50 MPa) and higher surface temperatures ($T_s \leq$ 600–1000 K, internal heating and yield dependent). Once the system transitions to a stagnant-lid through surface temperatures effects, the planet will likely remain stagnant, indicating that all of this space is bi-stable (as opposed to mono-stable mobile-lid states implied from Fig. 4A). A transition back to a mobile lid cannot be achieved by the surface temperature returning to its initial value alone, additional feedback effects, such as decreasing internal heating rates (or temperatures) may be required (Fig. 4A). Taking these feedback effects into account indicates that terrestrial planets have the ability to alternate between multiple tectonic states over giga-year timescales, and that a single expression of a planetary evolution pathway (e.g., stagnant/near stagnant to episodic to mobile) should not necessarily be expected, and could be highly misleading. Even if one specific pathway becomes unlikely, bi-stability does increase the potential for regime shifts in general, suggesting that evolutionary pathways with multiple tectonic regime shifts may well be more likely than evolutionary histories in

Figure 5. (A) Numerical simulations illustrating divergent tectonic states near a critical transition point in bi-stable regime space (modified after Lenardic et al., 2016a). An initial stagnant-lid (single plate) system is allowed to evolve into a mobile-lid by decreasing internal heating (from an initial high to a later intermediate stage – a proxy for system ageing, see Weller et al. (2015) for background and discussion of numerical approaches). A random perturbation in the internal thermal field of the model planet, with a maximum non-dimensional amplitude of 0.1 and a mean value such that the bulk system average temperature remains constant, leads to diverging stable system states when applied near a transition in tectonic behaviors, i.e., the experiments map two stable evolution paths that stem from a bifurcation point. High mobility indicates mobile-lid states (blue field), while low Mobility indicates stagnant-lid states (red field), where Mobility = Surface V_{rms}/Total System V_{rms}. The dark blue line indicates the original, unperturbed system evolution. The dashed purple line indicates the perturbed system response. Both cases result in long-lived stable states: mobile-lid for the original response, stagnant-lid for the perturbed response. (B) Numerical simulations similar to those of (A) except that the divergence in model paths is now initiated by a change in the surface temperature condition applied to the system (modified after Lenardic et al., 2016a). (C) Similar to (B) though a change in the surface temperature condition is applied at different times in the overturn event. Effects are similar to (B) except magnitude of perturbation required to change regimes increases as mobility increases. A new mode of mobility is identified for high levels of surface temperature.

which a planet remains in one tectonic mode over its geologically active lifetime.

5. Discussion and implications

For discussions of planetary evolution, the diagrams of Fig. 4 can be thought of as a dynamic landscape over which different evolutionary paths can be projected. As a thought experiment, a planet with abundant radiogenic heating will start in a stagnant-lid state for reasonable lithospheric yield strength values (black circle with red outline; Fig. 4A) at some initial time (t_0). As radiogenic heating decreases, this hypothetical planet moves towards the critical internal heating rate that allows for a possible regime change (dashed critical line Fig. 4A), if conditions would favor such a change (e.g., Lenardic et al., 2016a). For a convective system that approaches a bifurcation point, very small internal or external perturbations allow for distinct tectonic states to emerge from an identical starting state (t_1; Fig. 4A). An example of this is illustrated in Fig. 5A.

The numerical experiments of Fig. 5A for all intents and purposes are designed to look at the sensitivity of the system to low amplitude, transient noise near a critical point (e.g., transition, or bifurcation point). Here the term noise, defined as short-lived internal fluctuations, is used to encapsulate physical processes not explicitly accounted for in the model space. Near critical points, low amplitude, potentially infinitesimal, perturbations can allow for transitions in the global system (Thom, 1983; Arnold, 1986). Once a transition from a stagnant-lid regime to a plate tectonic-like regime is initiated feedback processes begin to develop in such a way that the stability of the new state is reinforced (e.g., through changes in internal thermal structure (Lenardic and Kaula, 1994; Bunge et al., 2001; Weller and Lenardic, 2016)), provided there is sufficient time for the feedback mechanisms to self-reinforce (the particulars of that time scale remain not fully mapped out at present; this is outside the scope of this paper).

For the experiment of Fig. 5A, the random perturbation is internal to the system. It is of interest to also explore the effects of an external perturbation, linked to long-lived climatic shifts (e.g., surface temperature fluctuations), proceeding and during a regime shift (Fig. 5B and C). When a surface temperature perturbation is applied near a bifurcation point (Fig. 5B), the amplitude of the surface perturbations needed to initiate a global transition are relatively small (on the order of 10 K). As the regime transition is allowed to progress, as in Fig. 5C, the system moves further towards its new state, and larger perturbations are required to cancel the feedback effects stabilizing the new regime. Eventually, the perturbations can no longer affect the transitioning regime, indicating that the system has moved well past the bifurcation point and the system has stabilized into a regime with resilient feedback mechanisms. Interestingly, this effect seems to allow for a new, high temperature mobile-lid regime to exist, one that operates at a higher level of mobility (the ratio of surface to bulk system velocity) than standard simulations would suggest (high surface temperature mobility; yellow-green curve Fig. 5C).

The particular evolutionary paths of planets can be complex and variable. A particular evolutionary path, as may be the case for the Earth, while allowed, is not the only path planets can follow. Continuing the thought experiment outlined in Fig. 4A, two otherwise similar bodies (e.g., Earth and Venus) can evolve differently as a result of relatively small variations in internal fluctuations, associated with chaotic mantle convection, sending them along different paths at a bifurcation point (the factors that can generate different chaotic variations between two systems, with identical evolving mean values, will be effectively stochastic). Neither body needs to be inherently unique in terms of major compositional differences and/or highly different initial water contents. As a consequence this indicates that plate tectonics may have occurred in Venus' past and it indeed may still be an allowed state for Venus at present (though it would potentially require a larger perturbation to (re-)initiate).

With this in mind, an appropriate question regarding the onset of plate tectonics in general may not be what initiates it but when in a planets development plate tectonics becomes feasible (this is the distinction between asking what initiates a transition versus asking what conditions at what particular evolutionary time allow for the potential of a transition at all). For Venus/Earth type systems, this can be approached from temperature arguments (e.g., recast Fig. 4A in terms of temperature, where the right side is 'hot', and the left side is 'cold'). In so doing, stagnant-lids that run hotter than their mobile-lid counterparts, may be viewed as thermally immature (e.g., that they retain more heat at the same stage of their evolution, or they plot to the right side of Fig. 4A), while mobile-lids running at lower internal temperatures may be thought of being thermally mature (e.g., less heat is retained so as to plot to the center/left side of Fig. 4A). In this view, planets are classified by where they fall in their bulk internal temperature evolution, as opposed to specific solar system ages.

In this scheme, mobile-lid activity with associated bi-stability, would be more likely in a planet's "middle age" (e.g., Fig. 4A). It would be natural to attempt to expand this to the newly discovered large mass terrestrial planets. It, however would be misleading to do so directly. With all things being held equal in terms of composition and heat source density, large terrestrial planets (e.g., putative 'super Earths') would have increased overall radiogenic heat production, and as a consequence temperatures. As a result, these larger planets would effectively be shifted to the right on Fig. 4A in temperature space (that is they would run hotter than their Earth or Venus counterpart for the same radiogenic proportions), indicating that while they may exist at the same absolute time (i.e., time since formation) they may effectively be operating at a different thermal time (temperature stage of evolution). Accordingly, mobile-lids and bi-stability would be a consequence of temporally "ancient", but thermally "middle aged" bodies.

The idea of a planetary evolution framework, incorporating bi-stability and thermal maturity, does suggest that the 'fundamental' question regarding the uniqueness of the Earth, as tied to plate tectonics, is more than a little misleading (and a bit of a red herring). It inherently assumes a set state that the Earth deviates from. When considering planets and planetary states, within the framework we have outlined, the differences between similar planets can become ones of time (again not actual age but an internal thermal time) and/or potentially associated with stochastic fluctuations. In this view, plate tectonics is a state of planetary evolution that many bodies have the potential to operate within at different times in their evolution. Whether or not they operate within this tectonic path may not be due to strictly deterministic reasons (e.g., the system is primed in some unique way), but instead could result from effectively random perturbations that initiate self reinforcing feedback processes that progressively allow a tectonic state to become locked in. This idea has been bolstered recently by the work of Wong and Solomatov (2016) in which they found the initiation time scale of lithospheric yielding, under favorable - steady state - conditions, is random, and can effectively span the lifetime of a planet. The question then of the timing of the onset of plate tectonics, in a general sense, can become somewhat unproductive (although for an individual planet such as the Earth, with a good enough geologic record, it may still be worthwhile to explore in order to understand its specific evolution). A more productive tact for considering a broad sweep of terrestrial planets (including the Earth) may be to move toward developing a better theoretical understanding of not only the feedback mechanisms that reinforce and stabilize regimes, particularly those of plate tectonics, but also their time evolution. These ideas illustrate the need to think in terms of a planetary evolution framework that moves toward probabilistic lines of attack, based on general physical principals as opposed to particular rheologies, to map the potential for any lid-state, let alone a plate tectonic one, for the large set of terrestrial exoplanets that have been, and continue to be, found. Studies of these far-flung and diverse bodies will in turn allow us to test ideas relating to the tectonic evolution of bodies within our own solar system, and in so doing demand at fundamental level a rethink of how habitable zones are defined (e.g., Venus is at the edge of the current habitable zone because it currently does not have liquid water, not because it is inherently incapable of having liquid water at present). Indeed the statistical nature of the framework we have laid out, with its potential that initially similar planets can evolve very differently, demands more than one planetary body for testing. Closer to home, in our solar system, it also shows how much added hypothesis testing could come from even modest improvements in observations that can constrain the geologic history of Venus.

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